EE 362: Signals and Systems Name:----------

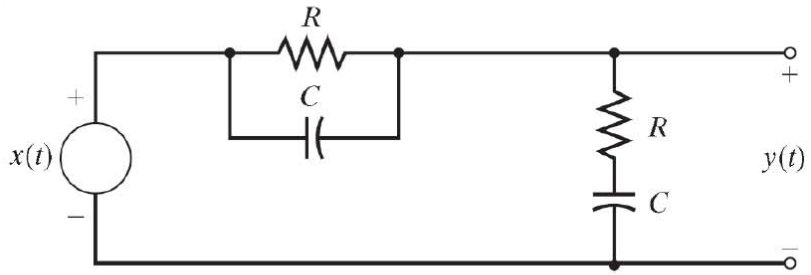
Spring 2019

Homework #12 (Due: Tuesday, April 30, 2019)

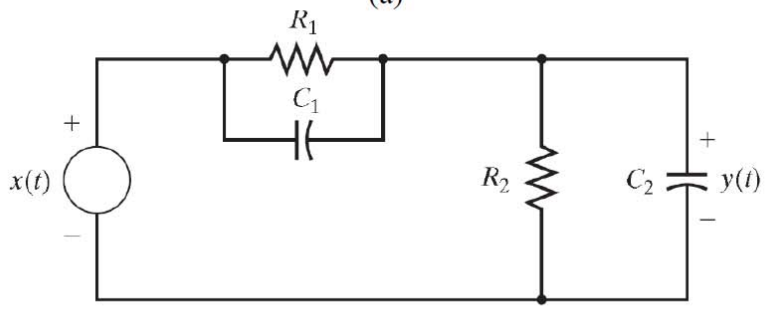
- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

In the following problems assume that all initial conditions are zero.

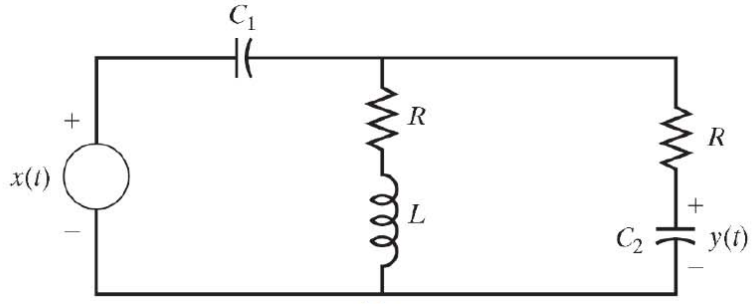
Find the transfer function of following systems/circuits. Parts (a) to (e) worth 3 points each and parts (f) and (g) worth 3.5 points each.



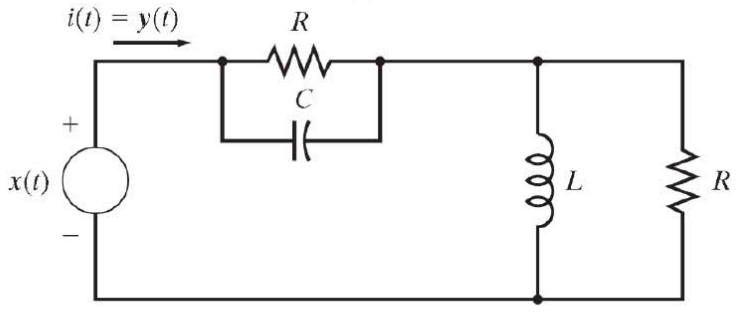
(a)



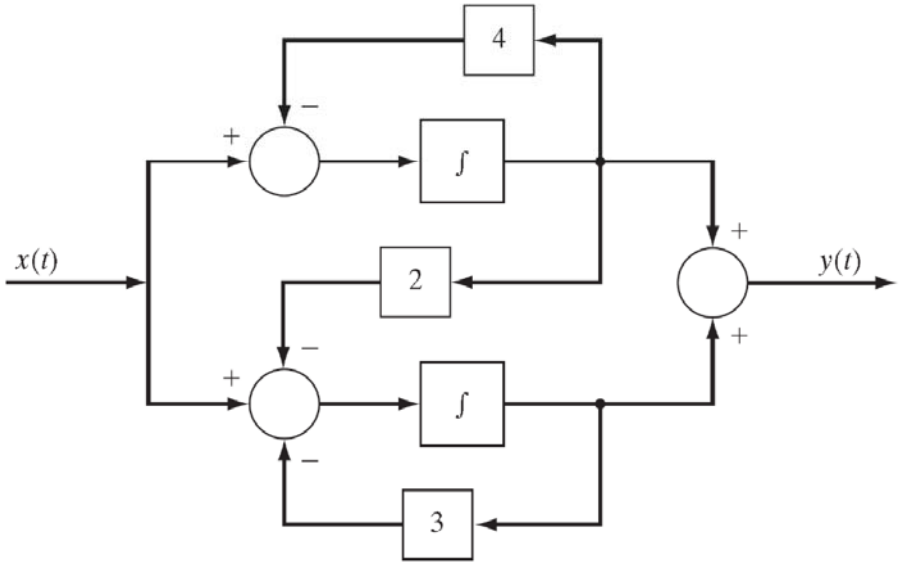
(b)



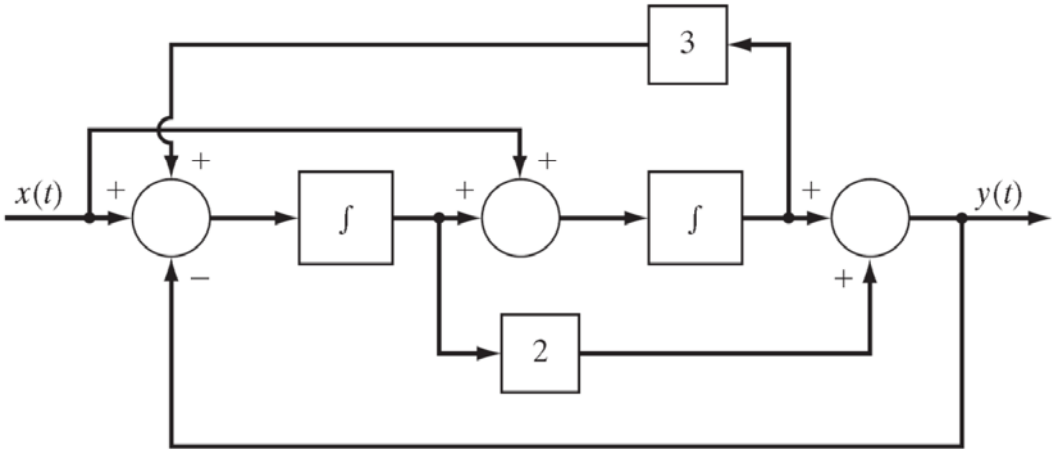
(c)



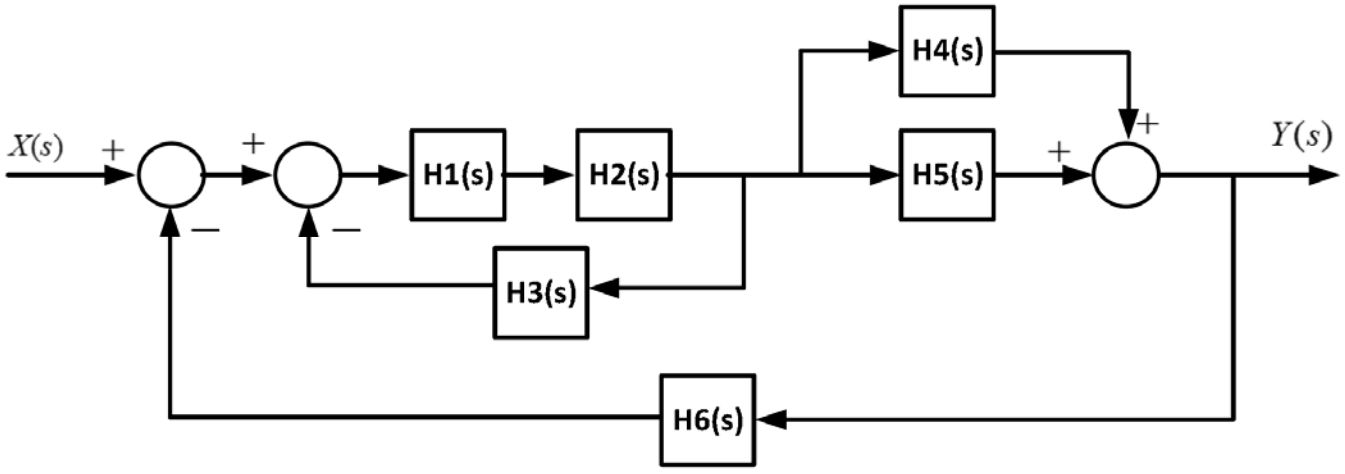
(d)



(e)

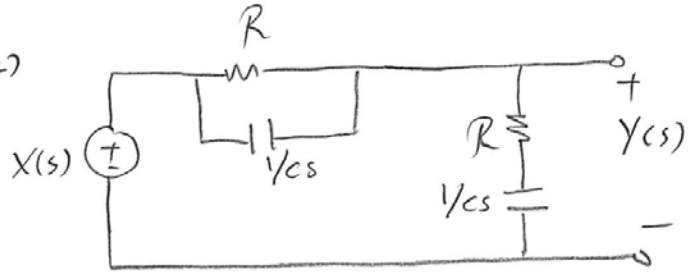


(f)



(g)

(a)



$$Y(s) = X(s)\left(\frac{R + 1/Cs}{R + 1/Cs + R \,//\, 1/Cs}\right) = \left(\frac{R + 1/Cs}{R + 1/Cs + \dfrac{R/Cs}{R + 1/Cs}}\right) X(s)$$

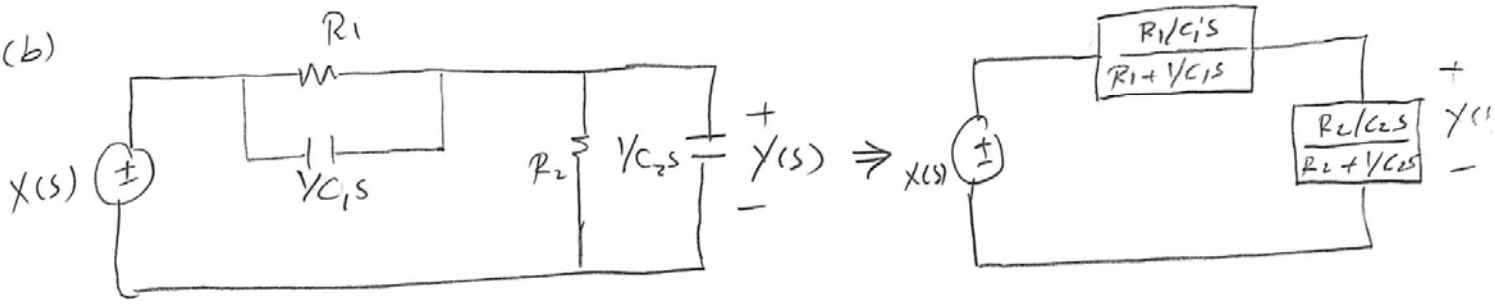
$$= \frac{(R + 1/Cs)^2}{(R + 1/Cs)^2 + R/Cs}\, X(s)$$

$$= \frac{(RCs + 1)^2}{(RCs + 1)^2 + RCs}\, X(s)$$

$$= \frac{R^2C^2s^2 + 2RCs + 1}{R^2C^2s^2 + 3RCs + 1}$$

$$\therefore \boxed{H(s) = \frac{R^2C^2s^2 + 2RCs + 1}{R^2C^2s^2 + 3RCs + 1}}$$

(b)



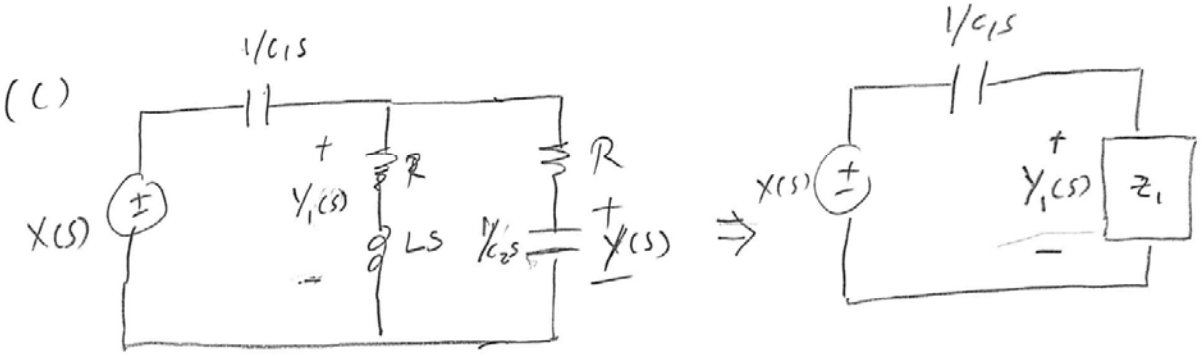
$$Y(s) = X(s)\left(\frac{\dfrac{R_2/C_2s}{R_2 + 1/C_2s}}{\dfrac{R_1/C_1s}{R_1 + 1/C_1s} + \dfrac{R_2/C_2s}{R_2 + 1/C_2s}}\right)$$

$$\therefore H(s) = \frac{(R_1 + 1/C_1 s)(R_2 + 1/C_2 s)}{(R_1 + 1/C_1 s)(R_2 + 1/C_2 s)} \left(\frac{\dfrac{R_2/C_1 s}{R_2 + 1/C_2 s}}{\dfrac{R_2/C_2 s}{R_2 + 1/C_2 s} + \dfrac{R_1/C_1 s}{R_1 + 1/C_1 s}} \right)$$

$$= \frac{\dfrac{R_2}{C_2 s}(R_1 + 1/C_1 s)}{\dfrac{R_2}{C_2 s}(R_1 + 1/C_1 s) + \dfrac{R_1}{C_1 s}(R_2 + 1/C_2 s)} \times \frac{C_1 C_2 s^2}{C_1 C_2 s^2}$$

$$\therefore \boxed{H(s) = \frac{R_2(R_1 C_1 s + 1)}{R_2(R_1 C_1 s + 1) + R_1(R_2 C_2 s + 1)}}$$

(c)



$$Z_1 = \frac{(R + Ls)(R + 1/C_2 s)}{R + Ls + R + 1/C_2 s} = \frac{(R + Ls)(RC_2 s + 1)}{C_2 s(2R + Ls) + 1}$$

$$\therefore Y_1(s) = X(s) \left(\frac{\dfrac{(R + Ls)(RC_2 s + 1)}{C_2 s(2R + Ls) + 1}}{\dfrac{1}{C_1 s} + \dfrac{(R + Ls)(RC_2 s + 1)}{C_2 s(2R + Ls) + 1}} \right)$$

$$= X(s) \left(\frac{C_1 s(R + Ls)(RC_2 s + 1)}{C_1 s(R + Ls)(RC_2 s + 1) + C_2 s(2R + Ls) + 1} \right)$$

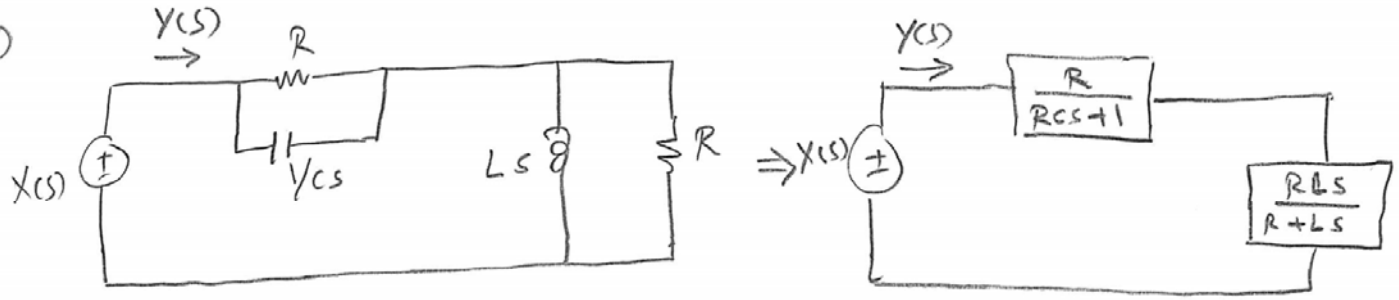
$$Y(s) = Y_1(s) \left(\frac{1/C_2 s}{R + 1/C_2 s} \right) = Y_1(s) \left(\frac{1}{1 + RC_2 s} \right)$$

$$Y(s) = X(s)\left(\frac{C_1 s(R+Ls)(RC_2 s+1)}{C_1 s(R+Ls)(RC_2 s+1)+C_2 s(2R+Ls)+1}\right)\left(\frac{1}{1+RC_2 s}\right)$$

$$= X(s)\left(\frac{C_1 s(R+Ls)}{C_1 s(R+Ls)(RC_2 s+1)+C_2 s(2R+Ls)+1}\right)$$

$$\therefore \boxed{H(s) = \frac{C_1 s(R+Ls)}{C_1 s(R+Ls)(RC_2 s+1)+C_2 s(2R+Ls)+1}}$$

(d)

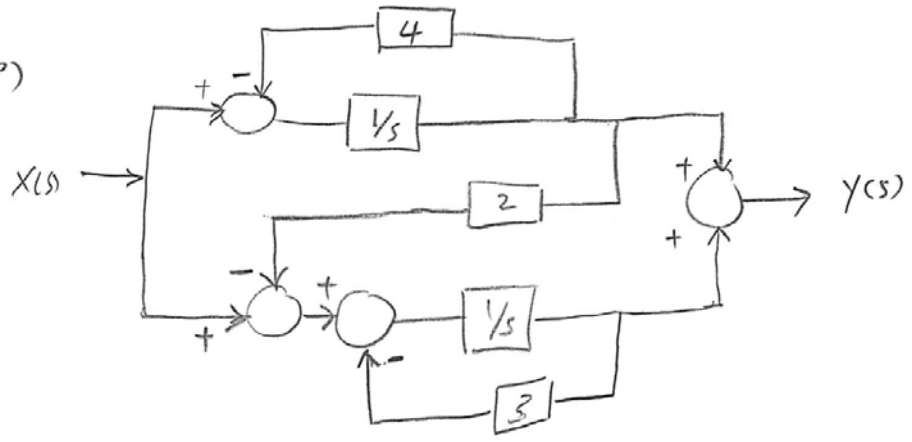


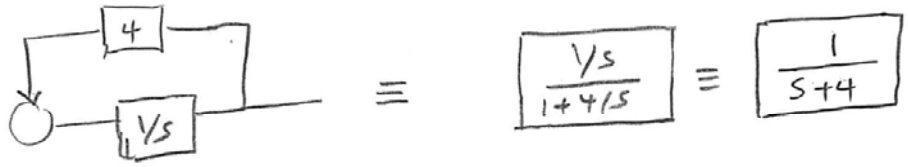
$$R \,//\, (1/Cs) = \frac{R/Cs}{R+\frac{1}{Cs}} = \frac{R}{RCs+1} \quad \& \quad R \,//\, Ls = \frac{RLs}{R+Ls}$$

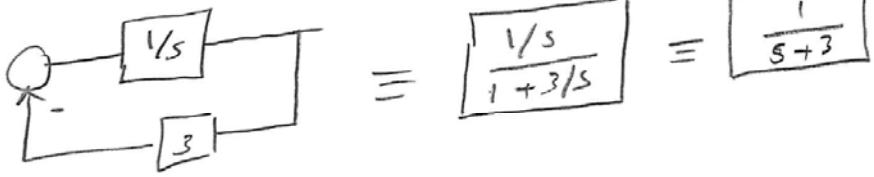
$$Y(s) = \frac{X(s)}{\frac{R}{RCs+1}+\frac{RLs}{R+Ls}} \Rightarrow H(s) = \frac{1}{\frac{R}{RCs+1}+\frac{RLs}{R+Ls}}$$

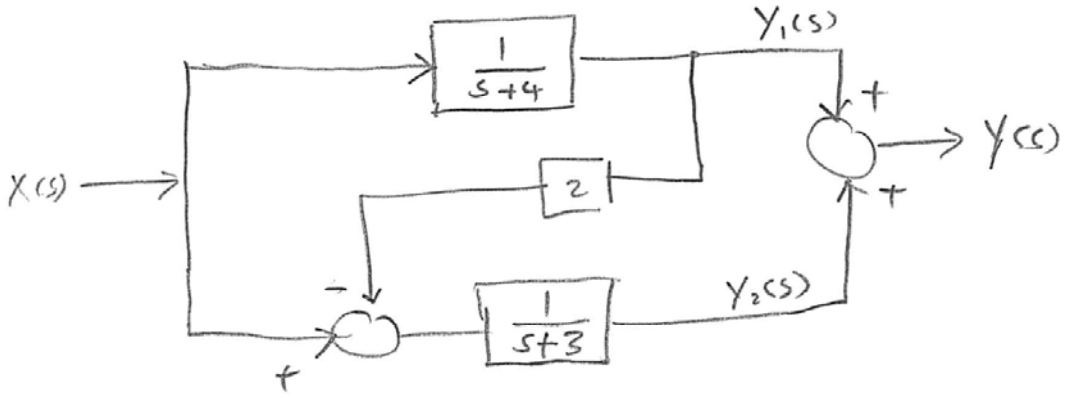
$$\therefore \boxed{H(s) = \frac{(RCs+1)(R+Ls)}{R(R+Ls)+RLs(RCs+1)}}$$

(e)









$$Y_1(s) = X(s)\left(\frac{1}{s+4}\right)$$

$$Y_2(s) = \left[X(s) - 2Y_1(s)\right]\left(\frac{1}{s+3}\right) = \left[X(s) - 2X(s)\left(\frac{1}{s+4}\right)\right]\left(\frac{1}{s+3}\right)$$

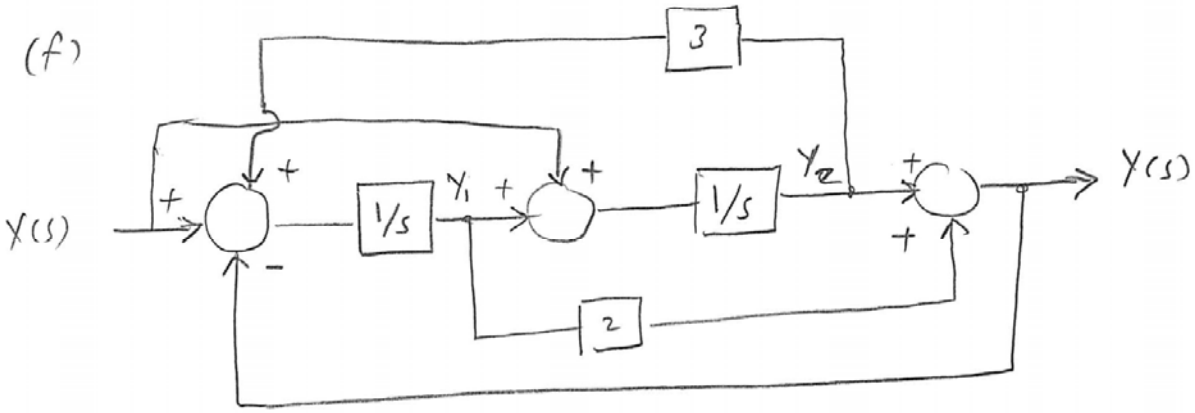
$$= X(s)\left[\frac{1}{s+3} - \frac{2}{(s+4)(s+3)}\right]$$

$$Y(s) = Y_1(s) + Y_2(s) = X(s)\left[\frac{1}{s+4} + \frac{1}{s+3} - \frac{2}{(s+4)(s+3)}\right]$$

$$= X(s)\left[\frac{(s+3)+(s+4)-2}{(s+4)(s+3)}\right] = X(s)\left[\frac{2s+5}{(s+4)(s+3)}\right]$$

$$\therefore \boxed{H(s) = \frac{2s+5}{(s+4)(s+3)}}$$

(f)



$$Y(s) = 2Y_1(s) + Y_2(s)$$

$$Y_1(s) = \left(\frac{1}{s}\right)\left(X(s) + 3Y_2(s) - Y(s)\right)$$

$$Y_2(s) = \left(\frac{1}{s}\right)\left(X(s) + Y_1(s)\right)$$

$$\therefore Y_1(s) = \frac{1}{s}\left(X(s) + 3\left(\frac{1}{s}\right)\left(X(s) + Y_1(s)\right) - Y(s)\right)$$

$$= \frac{1}{s}X(s) + \frac{3}{s^2}X(s) + \frac{3}{s^2}Y_1(s) - \frac{1}{s}Y(s)$$

$$\therefore Y_1(s)\left(1 - \frac{3}{s^2}\right) = \left(\frac{1}{s} + \frac{3}{s^2}\right)X(s) - \frac{1}{s}Y(s)$$

$$\therefore Y_1(s)\left(\frac{s^2-3}{s^2}\right) = \frac{s+3}{s^2}X(s) - \frac{1}{s}Y(s)$$

$$\therefore Y_1(s) = \frac{\frac{s+3}{s^2}}{\frac{s^2-3}{s^2}}X(s) - \frac{\frac{1}{s}}{\frac{s^2-3}{s^2}}Y(s) = \frac{s+3}{s^2-3}X(s) - \frac{s}{s^2-3}Y(s)$$

$$\therefore Y_2(s) = \frac{1}{s}X(s) + \frac{1}{s}\left(\frac{s+3}{s^2-3}X(s) - \frac{s}{s^2-3}Y(s)\right)$$

$$= \left(\frac{1}{s} + \frac{s+3}{s(s^2-3)}\right)X(s) - \frac{s}{s(s^2-3)}Y(s)$$

$$= \frac{s+1}{(s^2-3)}X(s) - \frac{1}{(s^2-3)}Y(s)$$

$$Y(s) = 2Y_1(s) + Y_2(s)$$

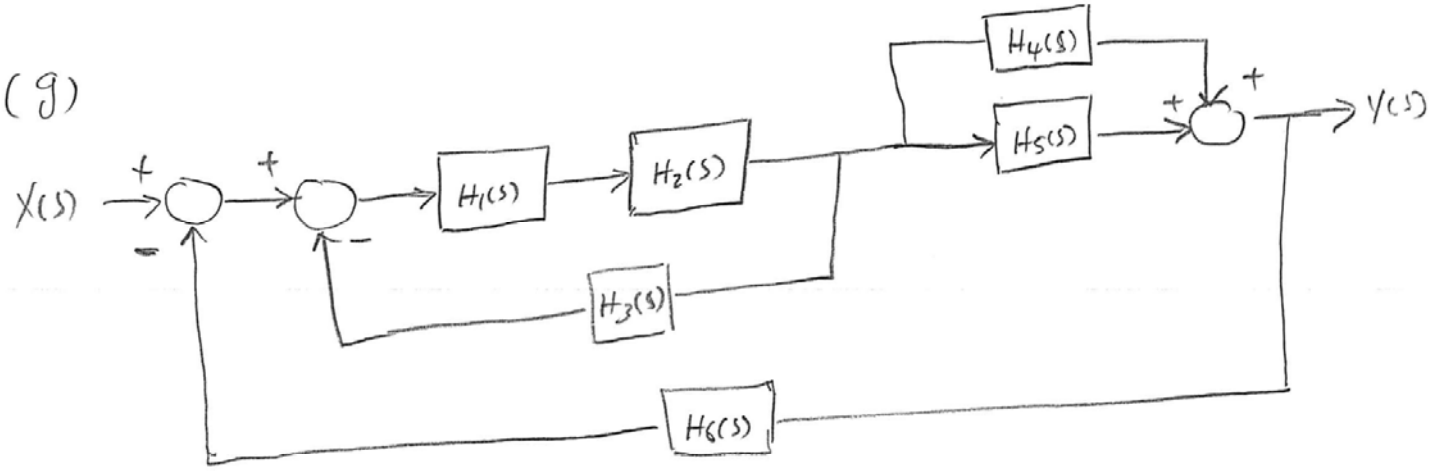
$$= 2\left(\frac{s+3}{s^2-3}X(s)\right) - \frac{2s}{s^2-3}Y(s) + \frac{s+1}{s^2-3}X(s) - \frac{1}{s^2-3}Y(s)$$

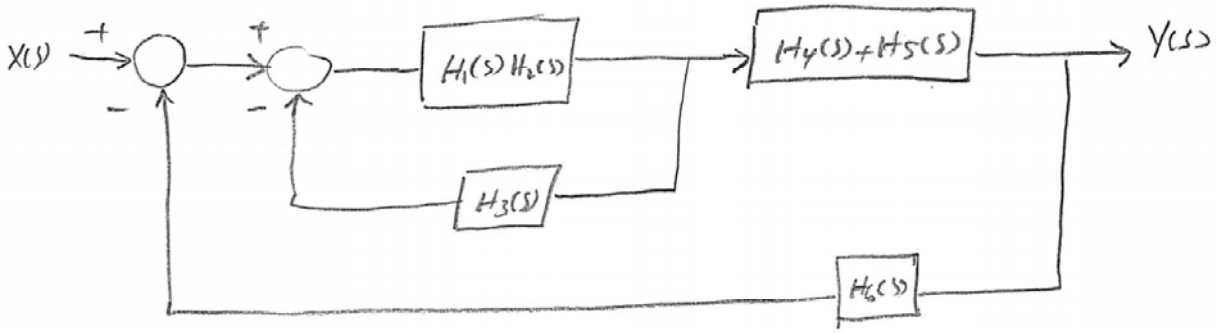
$$\therefore Y(s)\left[1 + \frac{2s}{s^2-3} + \frac{1}{(s^2-3)}\right] = \left[\frac{2(s+3)}{(s^2-3)} + \frac{s+1}{(s^2-3)}\right]X(s)$$

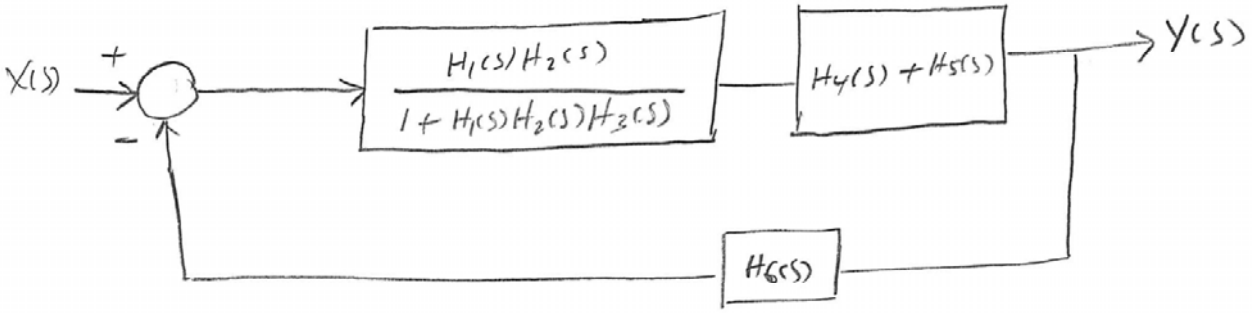
$$Y(s)\left[\frac{s^2-3+2s+1}{s^2-3}\right] = \left[\frac{2s+6+s+1}{s^2-3}\right]X(s)$$

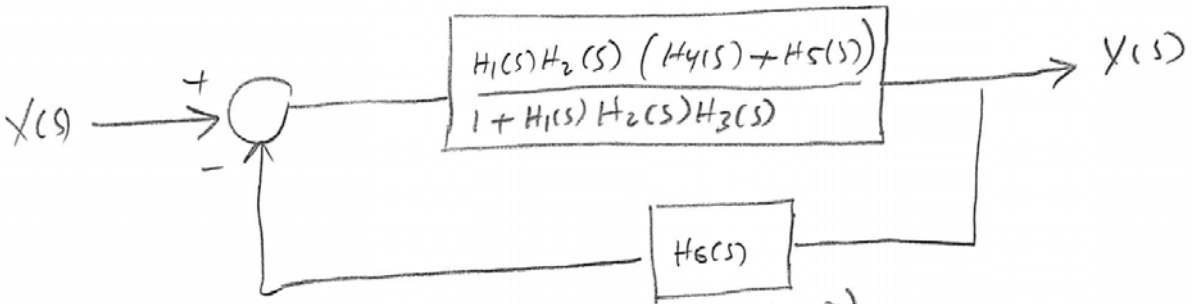
$$\therefore \boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{3s+7}{s^2+2s-2}}$$

(9)









$$\therefore H(s) = \frac{\dfrac{H_1(s)\,H_2(s)\,(H_4(s)+H_5(s))}{1+H_1(s)\,H_2(s)\,H_3(s)}}{1+\dfrac{H_1(s)\,H_2(s)\,H_6(s)\,(H_4(s)+H_5(s))}{1+H_1(s)\,H_2(s)\,H_3(s)}}$$

$$\therefore \boxed{H(s) = \frac{H_1(s)\,H_2(s)\,(H_4(s)+H_5(s))}{1+H_1(s)\,H_2(s)\,H_3(s)+H_1(s)\,H_2(s)\,H_6(s)\,(H_4(s)+H_5(s))}}$$